Senior Project

Delimit Service Scope of Power Plants by Weighted Voronoi Diagram

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Abstract The paper will present an application of Voronoi diagram on improving power distribution efficiency by delimiting geographically service scope of power plants in the U.S. A weighted 2D Voronoi diagram will be constructed based on factors such as the capacity of the power plants (the maximum electric output a power plant can produce), the location of the power plants, etc. Delimiting an appropriate service scope of each power plant is crucial for energy efficiency, because the longer the distance of transmission, the higher the energy loss would be. In the end, a Voronoi diagram will be constructed and visualized with the idealized service scope of each power plant.

Acknowledgments

Here, you can thank whoever you wish to thank.

Contents

1 Introduction

Electricity distribution and transmission efficiency has been a crucial consideration when constructing the electricity delivery system. A main factor that contributes to the inefficiency is energy loss during long distance transmission [Mar]. In the United States, modern electricity transmission systems have driven toward larger scale and more centralized electricity generation utilities, in contrast to local, small electric utilities that were dominant before the early 20th century [Uni]. That is, the electricity that flows to the consumer is usually generated from a distant power station and travels along the long transmission lines across cities, even states [Uni]. Although the centralized layout has increased supply reliability, the drawback is also salient – the transmission cost could be up to 30% more than the distributed layout [Mar]. "Line loss", the loss caused by the resistance when the electricity flows through the cable, contributes to the main energy loss, and the loss increases proportionally with the transmission distance, which can be up to $10\% - 20\%$ of the total electricity transmitted [Ele].

Therefore, an ideal electricity distribution model should allocate each demand point to the nearest power generator possible without exceeding each generator's maximum energy output, the capacity of the generator. To construct such a distribution model, a 2D weighted Voronoi diagram(VD) is applied to a dataset of over 8000 power plants in the United States, including each of their location and capacity. By VD's nature of capturing proximity, the territory is partitioned into regions such that all demand points within the same region are closer to a power plant than to any other power plants. We define such a region as the optimal service scope of a power plant, where the power plant will be the main the power source for all demand points within the region. By determining the optimal service scope of each power plant, the loss caused by long distance transmission could be reduced by having each power plant be the main power provider of each demand point within its optimal service scope. We will further introduce VD's properties and applications in the following section.

2 Background

2.1 Introduction to Voronoi Diagram

2.1.1 Voronoi Diagram and Properties

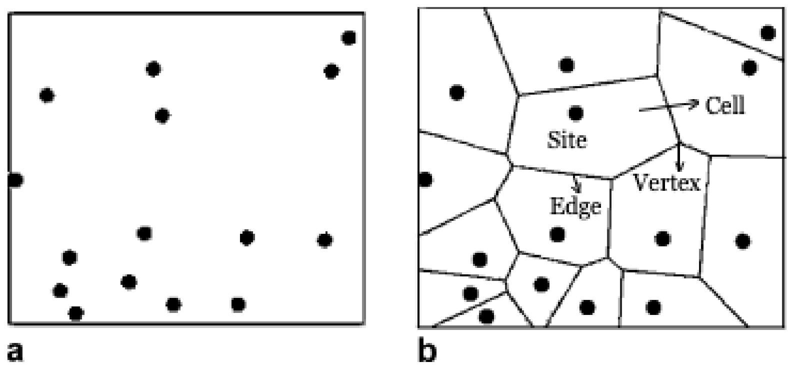


Figure 1: An example of VD creation [Kan08].

The traditional VD is a partition of plane based on a set of points. Those points are referred to as "sites" or "generators". Each site has one corresponding region called "cell". All the points in a cell are closer to the corresponding site than to any other site. The points that are equidistant from two sites form a Voronoi edge, and the points that are equidistant from more than two sites are Voronoi vertices. The following is some of the properties of VD:

1. All Voronoi regions are convex.

2. The generator g is the nearest generator point from point p if and only if the corresponding cell of g contains p.

3. Voronoi cells are mutually exclusive except for boundaries.

4. The union of all Voronoi cells covers the entire plane.

5. The straight-line dual graph of VD corresponds to Delaunay Triangulation, which will be introduced in the later section.

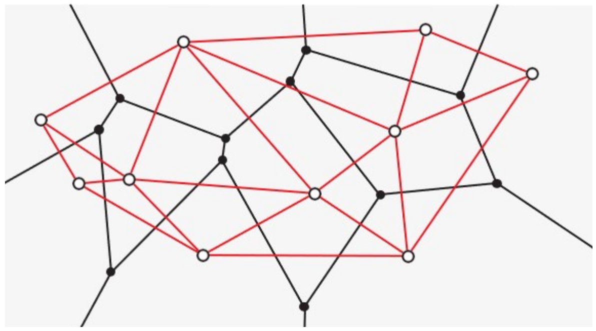


Figure 2: Dual of VD: connecting sites if they share a common Voronoi edge. The result is a Delaunay triangulation (in red) [DO11].

2.1.2 Delaunay Triangulation and Properties

Given a set of points S, a convex hull is outlined in blue as shown in graph below. A triangulation of S is a partition of the convex hull. Each subdivision (face) is a triangle, whose vertices are the points and does not contain other points.

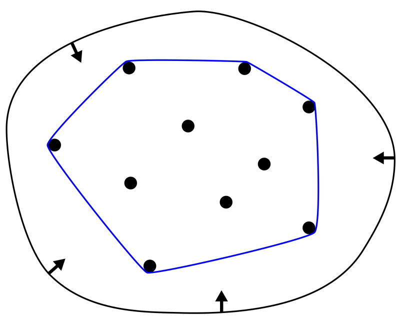


Figure 3: The convex hull of a point set: the region outlined by blue segments[Wik21a].

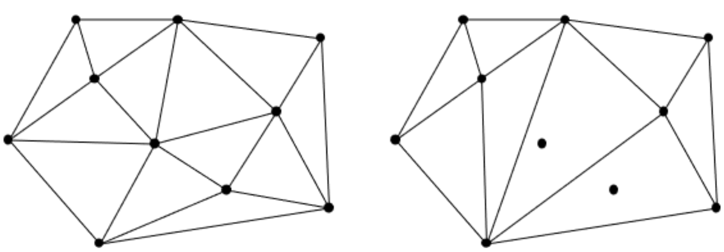


Figure 4: (a) Point set triangulation. (b) Not a triangulation [ETH13].

There are exponential number of triangulations given a point set. Delaunay triangulation (DT) is one special kind of triangulation where the circumcircle of each triangle does not contain other points, known as “the empty circle property”.

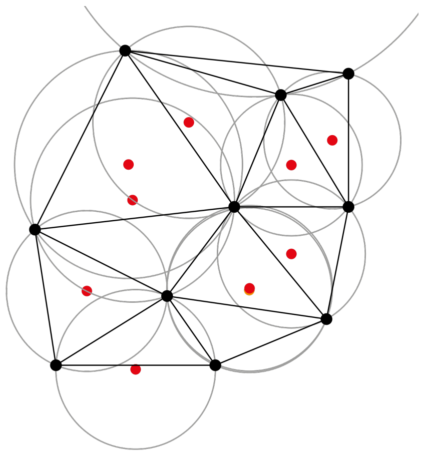


Figure 5: A DT with circumcircles(in grey) and circumcenters(red dots). Note that all circles are empty of points(black dots)[Wik21b].

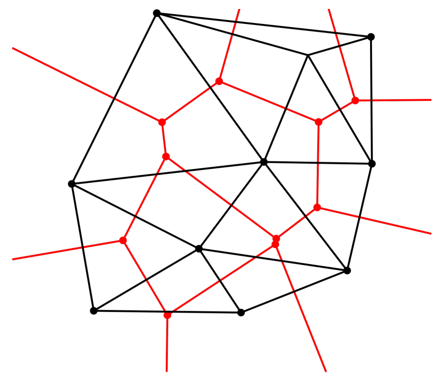


Figure 6: Connecting the centers of circumcircles produces a VD[Wik21b].

As mentioned above, a vital feature of DT is its duality with VD. Figure 2 and 6 shows the transformation from one to the other. “The one-to-one correspondence provided by duality” suggests that VD can be computed via DT[DO11]. In section 4, we will further discuss different algorithms of VD construction.

Another important property of DT is that it tends to avoid “skinny” triangles, which are triangles with one or two extremely acute angles and hence have a long, thin shape. This property makes DT more desirable in some circumstances. For example, in terrain construction, DT can better capture the feature of land surfaces such as ridges and valleys compared to other triangulations[DO11].

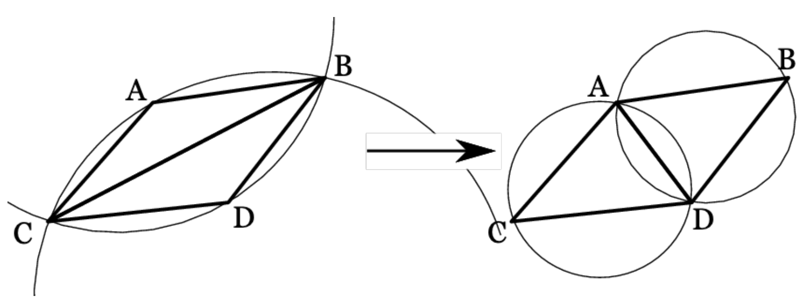


Figure 7: (a) Non DT has long, thin triangles (b) DT has “fat” triangles [Flo16].

2.2 Algorithms for Computing Voronoi Diagrams

One way of constructing the VD is via DT. One computes DT first and transforms the solution to its dual VD. A straight forward DT solution is the flip algorithm. It starts with a random triangulation and keeps “flipping” edge, as shown in figure 7, until all the triangles satisfy empty circle property. Other DT algorithms includes Bowyer-Watson algorithm, DeWall algorithm, and Quickhull algorithm.

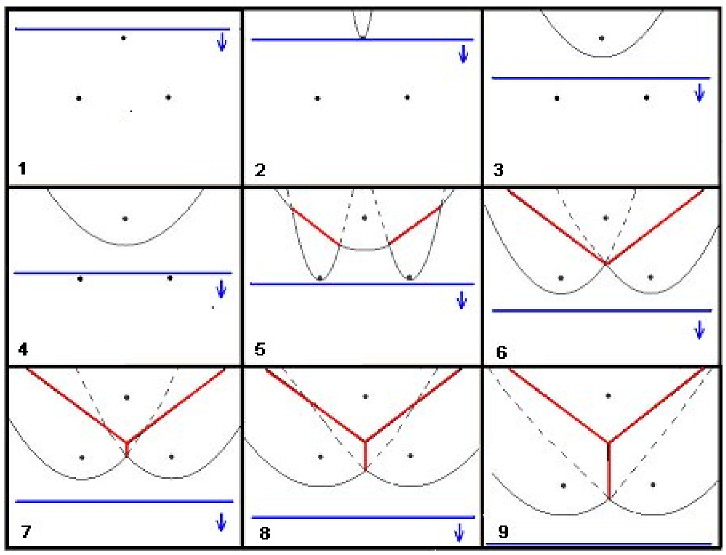


Figure 8: A demo of sweep line algorithm showing how VD (in red) is traced out by break points[Tah06].

For direct construction of VD, there are various approaches, including the incremental approach, the divide-and-conquer approach and Fortune’s algorithm (the sweep line algorithm). The incremental approach proposed by Peter Green and Robin Sibson in 1977 constructs the diagram by including one site a time to the current diagram and form a new cell, until all sites are included. The time-complexity is $O(n^2)$ [DO11]. The divide-and-conquer approach recursively divides the plane into two halves and construct the diagram for each half, and then combines the two diagrams together. The time complexity is $O(nlogn)$, which is more efficient but the combination step “is complex and hard to implement”[For86]. Fortune’s sweep line has the same time complexity as the divide-and-conquer approach but a less complicated implementation. The algorithm maintains a horizontal line sweeping from top of the plane to the bottom, which is defined as "sweep line". The sweep line and the sites above co-determine a series of parabolas. The union of the parabolas forms the "beach line", which changes as the sweep line moves. The changes on beach line give information on Voronoi egdes and vertices. The breakpoints on the beach line trace out the Voronoi edges. The disappearance of an parabola generates a Voronoi vertex. When the sweeping ends, the VD is produced[For86].

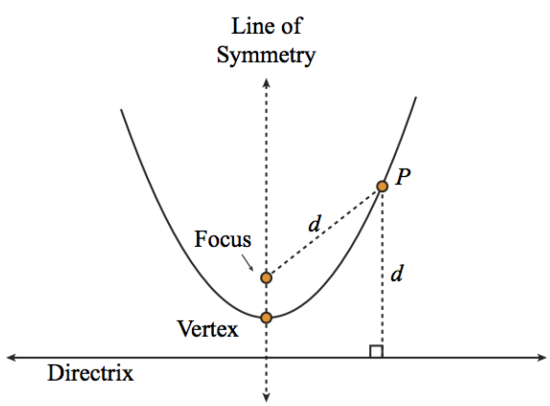


Figure 9: A point (focus) and a line (directrix) defines a parabola[par].

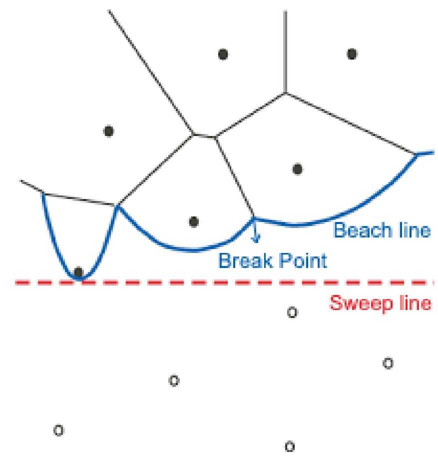


Figure 10: Static visualization of Fortune's algorithm.

2.3 Applications of Voronoi Diagram

The simplicity and elegance of concept and its nature of capturing proximity has made VD widely used and applied to solve various types of problems[DO11]. In particular, VD has major applications in solving proximity, similarity and coverage problems. For proximity, VD helps to find the nearest facilities such as hospitals or post office[Tra], and it also contributes to solving shortest-path finding problems[ADMS13] and k-closest-neighbor problem[KS04]. For similarity, VD has been applied to objects matching[DMDG15], categorization[SPR19] and clustering[KRE06]. For coverage, VD has been used for facility layout planning[LSL16] and optimize the service scope of facilities [PZ].

In addition, VD has extensive application in a variety of disciplines including archaeology, meteorology, biology, epidemiology, etc. In archaeology, VDs are often used to define the domain of influence of neolithic clans or hillforts [OBSC09]. In biology, Voronoi tessellation helps to model the 3D bone microstructure [LLK$^+$12], contributing to the measurement of physical constraints of body tissues. In meteorology, VDs, referred to as Thiessen polygon [Sch98], are used to compute average areal rainfall of a catchment from discrete data collected by rain gauges. In epidemiology, a famous application is the 1854 London Broad Street cholera outbreak[UCL]. A VD is constructed to accurately delineate the region where the most deaths clustered and successfully identified the source of infection[UCL].

Moreover, various generalizations and extensions have been developed to resolve more complex problems or better adapt to real-world situations [OBSC09]. For example, the high-order VD(or k-order VD) has more than 1 points that constitute of a generator[OBSC09]. The k-order VD has been applied to facility location problems in terms of determining the critical k nearest facilities[OBSC09]. Another important generalization is the weighted VD. The traditional VD implicitly assumes that sites are equally weighted. The weighted VD takes the variation in the properties and attributes of sites into account, which could be more suitable for practical applications [FM18] such as economic markets modeling, delivery system design, functional territories for facilities, etc.[OBSC09].

3 Related Work

There is a large body of work using VD to compute spatial coverage[OBSC09]. We will focus on discussing studies that use weight VD to solve coverage problems because which is the approach we will be using ourselves. Weighted VD has been commonly to resource allocations and facility layout design. For example, a study on determining ecosystem service scope [PZ] compares the accuracy of traditional VD with the weighted VD. Since ecosystem service value is a crucial indicator of the strength of ecosystem and varies from region to region, the weighted diagram incorporates ecosystem service value and results in a more scientific service area delineation than the traditional approach does. The weighted VD has also been applied in a study of modeling water distribution networks [GK12]. The objective is to assign water demand points to the closest water sources. Being aware of the geographical unevenness, a weighted VD is constructed with an additive constant on obstacles and boundaries that could not be passed by distribution pipes. In addition, weighted VD has also been used to optimize the layout of urban system [XSM+10] and rural settlements[WLX+20], to choose a new site for chain stores[LSL16], etc.

Moreover, the weighted VD has also been largely applied to emergency modeling and rescue route optimization, which usually adapts to the real-world circumstances better than the traditional VD[FM18]. For example, a study in delimiting the service area of fire emergency stations has the fire risk index as the weight incorporated into distance calculation. In addition, instead of the Euclidean distance, Manhattan distance is adopted to mimic the actual rescue routes[YCC+]. Another study on drone delivery of EMS from hospitals compares the traditional VD with the weighted VD that incorporates factors such as wind magnitude and obstacles. As a result, critical amount of response time, which is the most important measurement for delivery efficiency, could be saved by using the weighted VD [FM18]. Similarly, in a study on locating distribution center of disaster supplies, a weighted VD is applied to maximize the total coverage area of the distribution centers to produce a quick response time and minimize human suffering[YJU12] Moreover, weighted VD also helps with building an online map-based emergency management support system[LLTB11].

Thus, from the few studies discussed above, we see that weighted VD has been widely applied to solving coverage problems, which can compensate for spatial unevenness and produce more scientific results.

In this paper, we are going to apply a weighted VD on delimiting the service scope of power plants in the United States in order to minimize the energy loss in transmission. The weight is introduced to reflect the variation in power plant capacity. Though we have not found any direct VD application on service scope delimitation for power plants, there is a relevant study on applying weighted VD to determining new substation sites, substation capacity and service scope to minimize the overall annual cost [YF]. The weight has been used to reflect the variance in load distribution and capacity. The following is the major differences between our study and theirs:

1. In our study, the object is power plants which generate electricity, whereas their object is substations which distribute electricity.

2. Our study determines the service scope of located power plants, whereas they determines the location of new substations and their capacities.

3. Our objective is to reduce energy loss in transmission, and theirs is to reduce the overall substation operation cost.

Therefore, to our knowledge, this is the first attempt to reduce electricity loss in transmission by delimiting service scope for power plants. We have not found the information of the existing service scope of power plants within the U.S., so we are not able to compare our design with the existing service scope. We are also aware that the existing service scope may have taken other factors such as terrain, population density into consideration, whereas our model is idealized, only focusing on minimizing transmission loss irrespective of other practical issues.

4 Methodology

In order to determine each power plant's optimal service scope, we construct and compare the following two diagrams:

1. A traditional VD, where each Voronoi site represents a power plant. The distance measure used is the normal Euclidean distance.

2. A weighted VD, where each Voronio site has a radius of the square root of capacity. The distance measure used is power distance.

4.1 Data Description and Preprocessing

The data used for the application is obtained from a comprehensive, open source database of the power plants all over the world. The database is developed by World Resources Institute (WRI) in partnership with Google Earth Outreach and multiple authoritative organizations and it mainly draws upon trusted sources such as national governments and other official sources, integrated with a small portion of crowdsourced data such as satellite images for accuracy improvement [BFH+]. More importantly, among all current existing databases on this topic, it's the only one "truly comprehensive and fully accessible"[BFH+] with all sources traceable to publicly available webpages, and is still under maintenance. The dataset in CSV form can be accessed at https://www.kaggle.com/eshaan90/global-power-plant-database. We adapt the latest official release, the release of 2018.

The dataset contains over 28,500 power plants in 164 countries, representing about 80% of the world's capacity[BFH+]. The set of attributes of power plants covers identification information (name and unique identifier), geographic information (country, longitude and latitude in WGS84 standard), electricity generation data (generation capacity in megawatts, actual generation for each year from 2013 to 2016, estimated annual electricity generation), and other information including owner of the plant, year of operation, attribution of data sources.

We leverage Python pandas library for exploratory data analysis and wrangling. Since our study object is the power plants within the United States, we only keep the dataset entries that are within the U.S. The processed dataset has 8119 instances with 5 attributes: ID, name, longitude, latitude and capacity. Through the data profiling, there are no missing data, and have reasonable range of numbers. The electrical generation capacity ranges from 1 to 6800 megawatts, the longitude ranges from -171.71 to 144.90, the latitude ranges from

13.30-71.29. All figures are within reasonable ranges.

4.2 Tools and Packages

We construct the traditional VD by using D3-Voronoi package and visualize by Google Maps JavaScript API. D3-Voronoi is a package written in JavaScript for computing 2D-VD. We choose D3-Voronoi because it implements Fortune's algorithm and also has decent visualization (e.g. supports zoom transitions)[Riv]. The shortcoming is that the package itself does not support constructing more complex VD, including weighted VD, but it is satisfactory enough for constructing the traditional VD. Google Maps JavaScript API supports different types of map creation(roadmap, satellite, hybrid, and terrain), which are able to display on web pages and mobile devices [Goo].

For weighted VDs, the diagram is computed by using the Computational Geometry Algorithms Library (CGAL), which is a C++ library of robust geometry algorithms but has little built-in visualization support. Then, we use Processing, a graphical library and IDE, to visualize the diagram computed by CGAL. We use Google Static Map API to obtain static map image as the canvas for drawing.

5 Results and Analysis

In this section, we are going to compare the service scope delimitation results produced by traditional VD and by the weighted VD.

5.1 Traditional Voronoi Diagram and Visualization

Gaining insight from[Shi], diagrams are generated as Figure 9 and 10. As we can see, the territory of United States is partitioned into small cells by the red lines, and within each cell there is one black dot, which represents a power plant. The area covered by each cell is optimal service scope we have delimited for the power plant in the cell.

Figure 11: Service Scope Delimitation Visualization by Traditional VD

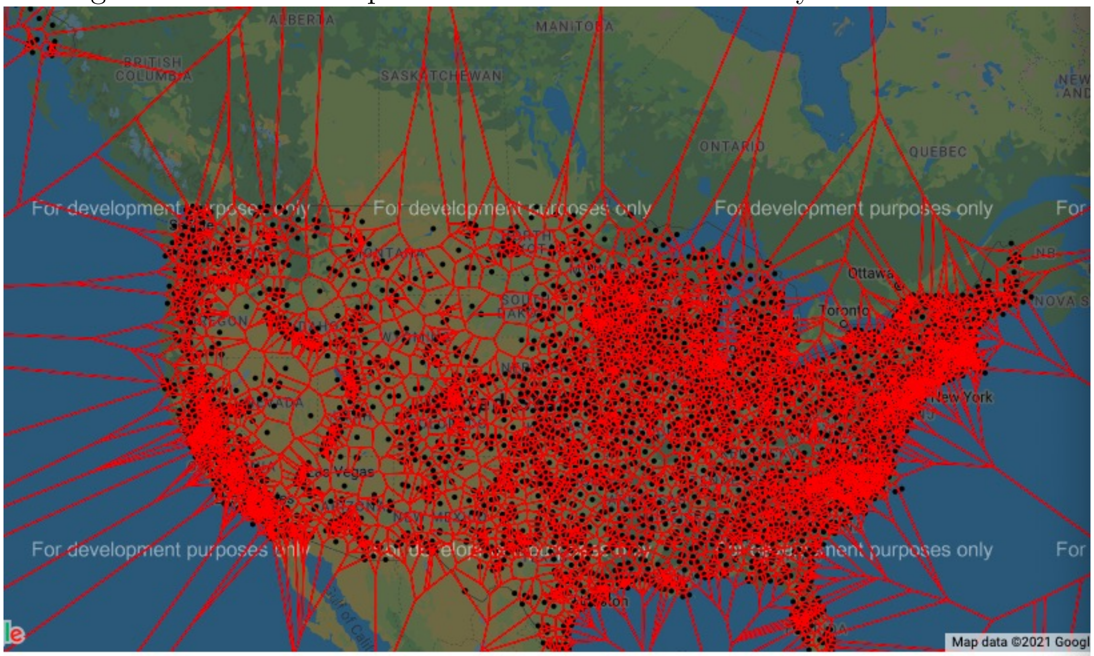
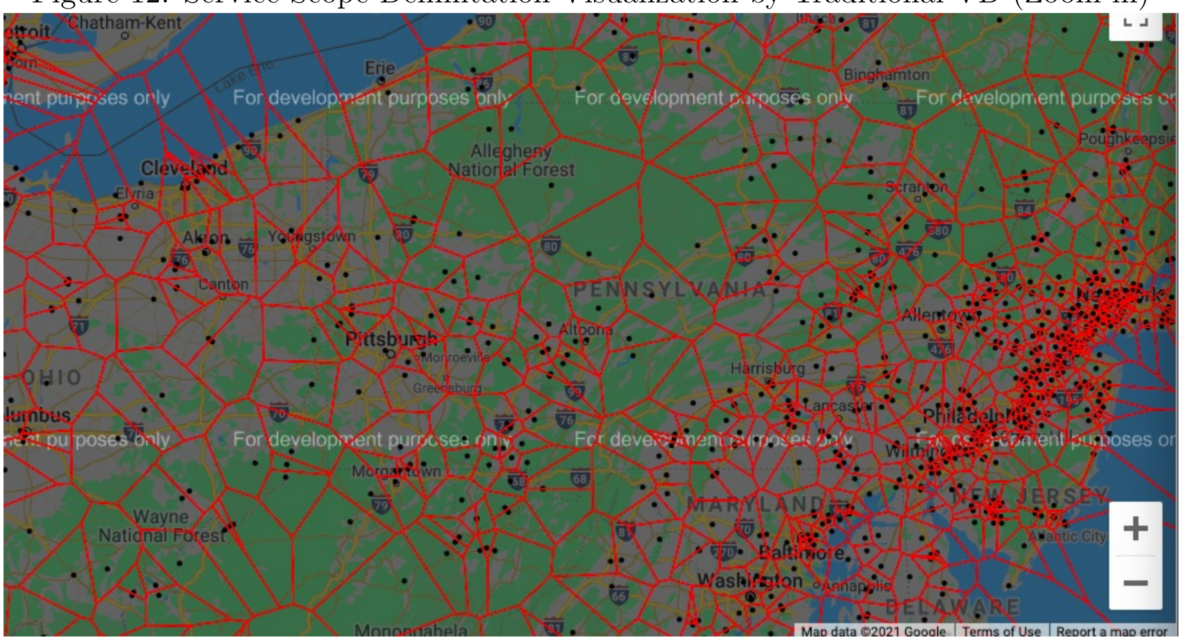


Figure 12: Service Scope Delimitation Visualization by Traditional VD (Zoom-in)



This approach simply "assigns" every point in the territory to the closest power plant in Euclidean distance, regardless the different attributes the power plants have in reality. In the next section, we will incorporate the capacity of each power plant as weight to the diagram construction.

5.2 Weighted Voronoi Diagram Approach and Visualization

6 Future Work

7 Summary

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Appendix