Senior Project

Delimit Service Scope of Power Plants by Weighted Voronoi Diagram

Kejing Wang

Yutong Wu

Advisor: Dianna Xu

Submitted in Partial Fulfillment of the Requirements of the BA in Computer Science

Bryn Mawr College

Spring 2021

Abstract The paper will present an application of Voronoi diagram to improve power distribution efficiency by delimiting geographically the service scope of each power plant in the U.S. A weighted 2D Voronoi diagram will be constructed based on factors such as the capacity (the maximum electric output a power plant can produce) and the location of the power plants. Delimiting an appropriate service scope of each power plant is crucial for energy efficiency, because the longer the distance of transmission, the higher the energy loss would be. In the end, Voronoi diagrams will be constructed and visualized with the idealized service scope of each power plant.

Acknowledgments

We would like to thank our advisor, Professor Dianna Xu. Professor Xu provided us with valuable guidance and advice, from the first steps to the result visualization.

We would also like to thank our peers for reviewing our drafts and giving constructive feedback.

Contents

List of Figures

1 Introduction

Electricity distribution and transmission efficiency has been a crucial consideration when constructing the electricity delivery system. A main factor that contributes to the inefficiency is energy loss during long distance transmission[Mar19]. In the United States, modern electricity transmission systems have driven toward larger scale and more centralized electricity generation utilities, in contrast to local, small electric utilities that were dominant before the early 20th century[Age21]. That is, the electricity that flows to the consumer is usually generated from a distant power station and travels along the long transmission lines across cities, even states[Age21]. Although the centralized layout has increased supply reliability, the drawback is also salient – the transmission cost could be up to 30% more than the distributed layout[Mar19]. "Line loss", the loss caused by the resistance when the electricity flows through the cable, contributes to the main energy loss, and the loss increases proportionally with the transmission distance, which can be up to $10\% - 20\%$ of the total electricity transmitted[Ele21].

Therefore, an ideal electricity distribution model should allocate each demand point to the nearest power generator possible without exceeding each generator's maximum energy output, the capacity of the generator. To construct such a distribution model, both the "traditional", unweighted Voronoi Diagram (VD) and weighted VD are applied to a dataset of over 8000 power plants in the United States, including each of their location and capacity. By constructing VD, the territory is partitioned into regions, and each region will be the optimal service scope of a power plant; that is, the power plant will be the main the power source for all demand points within the region. The traditional VD can minimize transmission loss by assigning each demand point to the closest possible power point; however, it does not consider practical factors such as the capacity of power plants, which can widely vary from one power plant to another. The weighted VD incorporates the capacity as weight to reduce potential short supply and avoid oversupply while still maintain VD's property of capturing proximity. We will further introduce VD's properties and applications in the following section.

2 Background

2.1 Introduction to Voronoi Diagram

2.1.1 Voronoi Diagram and Properties

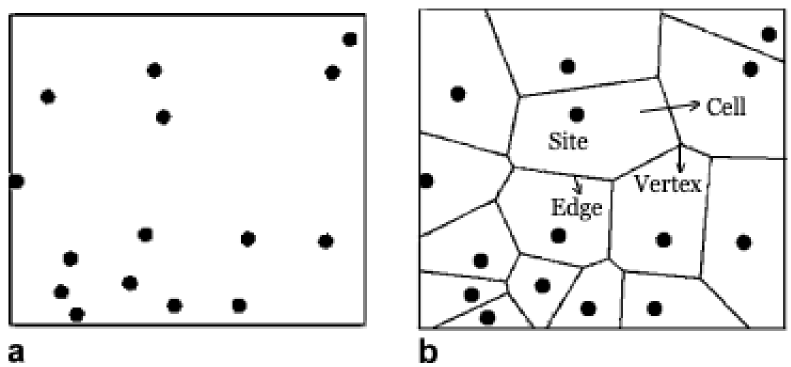


Figure 1: An example of VD creation[Kan08].

The (traditional) Voronoi Diagram (VD) is a partition of plane based on a set of points. Those points are referred to as "sites" or "generators". Each site has one corresponding region called "cell". All the points in a cell are closer to the corresponding site than to any other site. The points that are equidistant from two sites form a Voronoi edge, and the points that are equidistant from more than two sites are Voronoi vertices. The following is some of the properties of a VD[DO11]:

1. All Voronoi regions are convex.
2. Voronoi cells are mutually exclusive except for boundaries.
3. The union of all Voronoi cells covers the entire plane.
4. The generator g is the nearest generator point from point p if and only if the corresponding cell of g contains p.

5. For any two adjacent Voronoi sites that shares a common edge, the sites have the same distance to the edge.

6. The straight-line dual graph of a VD (see Figure 2) is a Delaunay triangulation, which will be introduced in the next section.

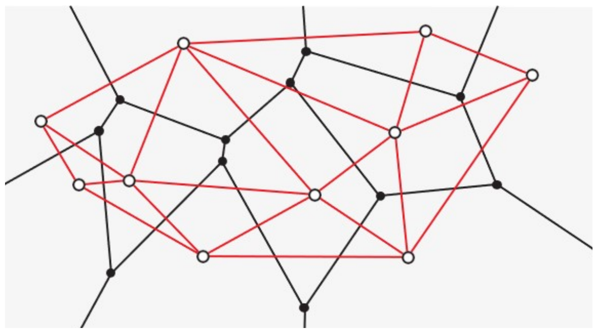


Figure 2: The straight-line dual graph of a VD is the graph produced by connecting every pair of Voronoi sites if they share a common Voronoi edge[DO11].

2.1.2 Delaunay Triangulation and Properties

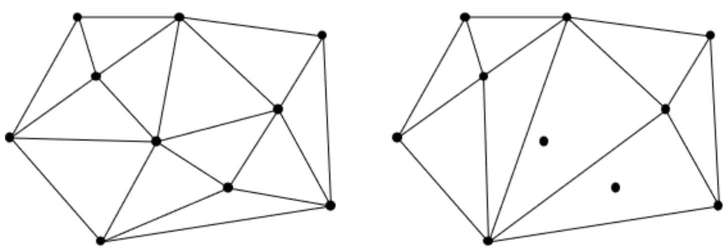


Figure 3: (a) A point set triangulation (b) Not a triangulation[Swi21].

A triagnuation of a point set S is a subdivision of the plane determined by a maximal set of non-crossing edges whose vertex set is S. There are exponential number of triangulations given a point set. Delaunay triangulation (DT) is one special kind of triangulation where the circumcircle of each triangle does not contain other points of S, known as "the empty circle property". Figure 4b demonstrates a non-Delanay triangulation where the circumcircle of T1 contains the point V3.

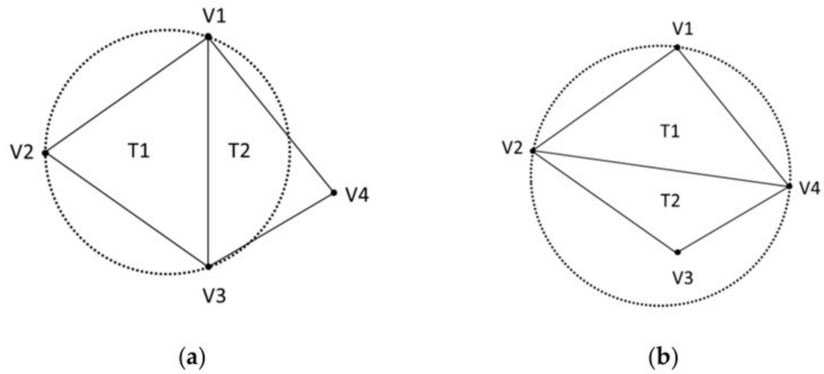


Figure 4: Comparison of Delaunay and non-Delaunay triangulations. (a) Delaunay triangulation (b) Non-Delaunay triangulation[Res21].

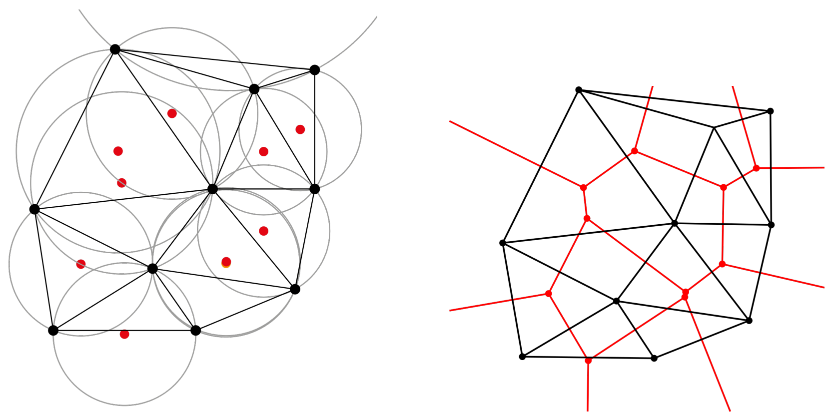


Figure 5: Connecting the circumcenters whose triangles share an edge produces a VD[Con21].

An important feature of Delaunay triangulation is its duality with VD. As mentioned in the previous section, given a VD defined on a set of sites S, the Delauany triangulation of S can be found by connecting any two sites $p, q \in S$ that share a Voronoi edge (see Figure 2). Reversely, given a Delaunay triangulation, a VD can be found by computing the circumcenters of all the triangles, and connecting any two circumcenters whose triangles share an edge (see Figure 5). "The one-to-one correspondence provided by duality" suggests that VD can be computed via Delaunay triangulation[DO11]. In section 2.2, we will further discuss different algorithms of VD construction.

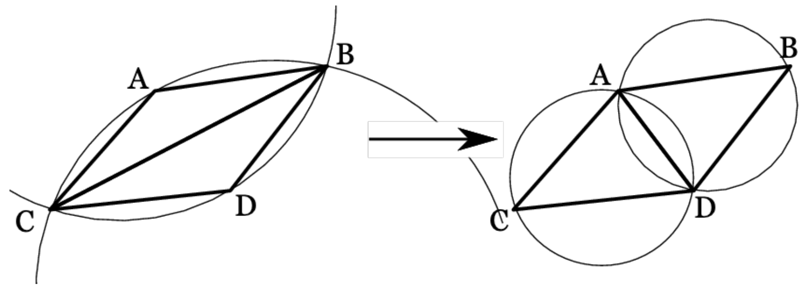


Figure 6: (a) Non DT has long, thin triangles (b) DT has “fat” triangles[Flo21].

Another important property of Delaunay triangulation (DT) is that it tends to avoid “skinny” triangles, which are triangles with one or two extremely acute angles and hence have a long, thin shape. This property makes DT more desirable in many applications. For example, in terrain construction, DT can better capture the feature of land surfaces such as ridges and valleys compared to other triangulations[DO11].

The generalized Delaunay triangulation is Regular triangulation, which is defined on a set of weighted points[Boi+02]. Regular triangulation is the dual of one type of weighted VD, the power diagram, which will be introduced in the section 4.

2.2 Algorithms for Computing Voronoi Diagrams

One way of constructing the VD is via Delaunay triangulation (DT). One computes DT first and transforms the solution to its dual VD. A straight forward DT solution is the flip algorithm. It starts with a random triangulation and keeps “flipping” edge, as shown in Figure 6, until all the triangles satisfy the empty circle property[DO11]. Other DT algorithms includes Bowyer-Watson algorithm, DeWall algorithm, and Quickhull algorithm[OBS09].

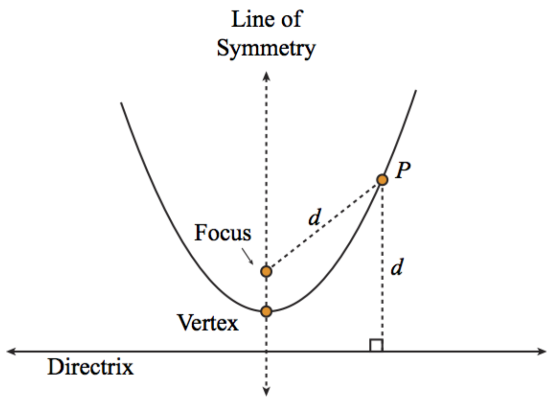


Figure 7: A point (focus) and a line (directrix) defines a parabola. Any point on the parabola is equidistant from the focus and from the line[Shm21].

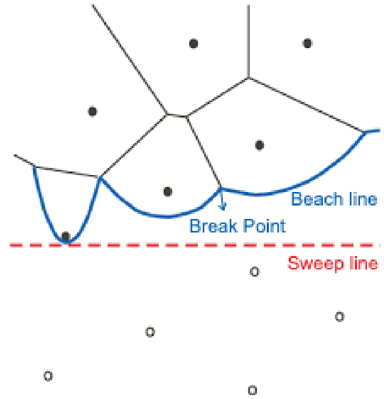


Figure 8: Static visualization of Fortune's algorithm[JRG11].

For direct construction of VD, there are various approaches, including the incremental approach, the divide-and-conquer approach and Fortune's algorithm (the sweep line algorithm). Proposed by Peter Green and Robin Sibson in 1977, the incremental approach adds one more site a time to the current diagram and forms a new cell, until all sites are included. The time-complexity is $O(n^2)$[DO11]. The divide-and-conquer approach recursively divides the plane into two halves, constructs the diagram for each half, and then combines the two diagrams together. The time complexity is $O(nlogn)$, which is more efficient but the combination step "is complex and hard to implement"[For86]. Fortune's sweep line algorithm has the same time complexity as the divide-and-conquer approach but a less complicated implementation. The algorithm maintains a horizontal line sweeping from top of the plane to the bottom, which is defined as "sweep line". The sweep line and the each site above it co-determine a parabola (see Figure 7). The opening of the parabola keep getting wider when the sweep lines moves away from the site. Hence, the union of all the parabolas, called the "beach line" keeps changing as the sweep line moves (see Figure 8). The changes on beach line provide information on Voronoi egdes and vertices (see Figure 9). The breakpoints on the beach line trace out the Voronoi edges. The disappearance of an parabola generates a Voronoi vertex. When the sweeping ends, the VD is produced[For86].

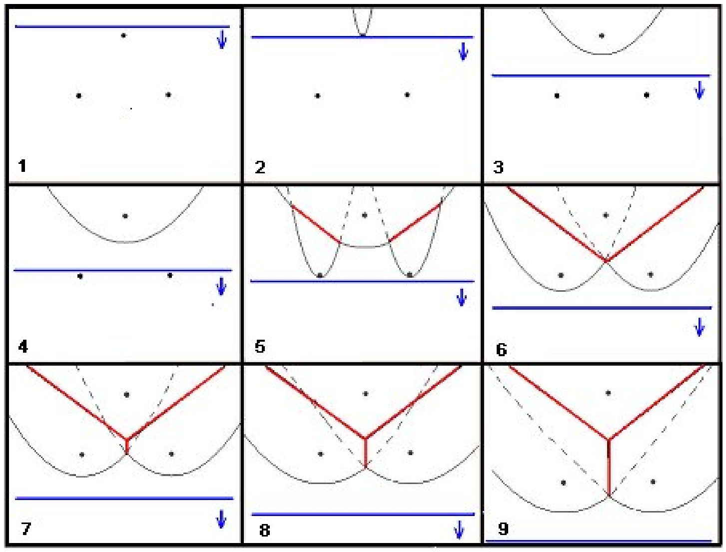


Figure 9: A demo of sweep line algorithm showing how VD (in red) is traced out by break points[Tah06].

2.3 Applications of Voronoi Diagram

The simplicity and elegance of concept and its nature of capturing proximity has made VD widely applied to solve various types of problems[DO11]. In particular, VD has major applications in solving proximity, similarity and coverage problems. For proximity, VD helps to find the nearest facilities such as hospitals or post office[Tra21], and it also contributes to solving shortest-path finding problems[Ale+13] and k-closest-neighbor problem[Kol04]. For similarity, VD has been applied to objects matching[Das+15], categorization[SPR19] and clustering[KRE06]. For coverage, VD has been used for facility layout planning[LSL16] and optimization of facilities' service scope[ZJC18].

In addition, VD has extensive application in a variety of disciplines including archaeology, meteorology, biology, epidemiology, etc. In archaeology, VDs are often used to define the domain of influence of neolithic clans or hillforts[OBS09]. In biology, Voronoi tessellation helps to model the 3D bone microstructure[Li+12], contributing to the measurement of physical constraints of body tissues. In meteorology, VD, referred to as Thiessen polygon[Sch98], is used to compute average areal rainfall of a catchment from discrete data collected by rain gauges. In epidemiology, a famous application is the 1854 London Broad Street cholera outbreak[Epi21]. A VD is constructed to accurately delineate the region where the most deaths clustered and helps to successfully identify the source of infection[Epi21].

Moreover, various generalizations and extensions have been developed to resolve more complex problems or better adapt to real-world situations[OBS09]. For example, the high-order VD (or k-order VD) has more than one point that constitute of a generator[OBS09]. The k-order VD has been applied to facility location problems in terms of determining the critical k nearest facilities[OBS09]. Another important generalization is the weighted VD. The "traditional" VD that we have dicussed so far implicitly assumes that sites are equally weighted. The weighted VD takes the variation in the properties and attributes of sites into account, which could be more suitable for practical applications[FM18] such as economic markets modeling, delivery system design, functional territories for facilities, etc.[OBS09].

3 Related Work

There is a large body of work using VD to compute spatial coverage[OBS09]. We will focus on discussing studies that use weighted VD to solve coverage problems because this is the approach we will be using ourselves.

Weighted VD has been commonly applied to resource allocations and facility layout design. For example, a study on determining ecosystem service scope[ZJC18] compares the accuracy of traditional VD with the weighted VD. Since ecosystem service value is a crucial indicator of the strength of ecosystem and varies from region to region, the weighted diagram incorporates ecosystem service value and results in a more scientific service area delineation than the traditional approach does. The weighted VD has also been applied in a study of modeling water distribution networks[GK12]. The objective is to assign water demand points to the closest water sources. Being aware of the geographical unevenness, a weighted VD is constructed with an additive constant on obstacles and boundaries that could not be passed by distribution pipes. In addition, weighted VD has also been used to optimize the layout of urban system[Xin^{+}10] and rural settlements[Wan^{+}20], to choose a new site for chain stores[LSL16], etc.

Moreover, the weighted VD has also been widely applied to emergency modeling and rescue route optimization, which usually adapts to the real-world circumstances better than the "traditional" VD[FM18]. For example, a study in delimiting the service area of fire emergency stations has the fire risk index as the weight incorporated into the distance calculation. In addition, instead of the Euclidean distance, Manhattan distance is adopted to mimic the actual rescue routes[Yu^{+}20]. Another study on drone delivery of EMS from hospitals compares the traditional VD with the weighted VD that incorporates factors such as wind magnitude and obstacles. As a result, critical amount of response time, which is the most important measurement for delivery efficiency, could be saved by using the weighted VD[FM18]. Similarly, in a study of locating distribution center of disaster supplies, a weighted VD is applied to maximize the total coverage area of the distribution centers to produce a quick response time and minimize human suffering[YJU12] Moreover, weighted VD also helps with building an online map-based emergency management support system[LLT11].

Thus, from the few studies discussed above, we see that weighted VD has been widely applied to solve coverage problems, which can compensate for variation of properties of sites and produce more scientific results.

In this paper, we are going to apply a weighted VD on delimiting the service scope of power plants in the United States in order to minimize the energy loss in transmission. The weight is introduced to reflect the variation in power plant capacity. Though we have not found any direct VD application on service scope delimitation for power plants, there is a relevant study on applying weighted VD to determining new substation sites, substation capacity and service scope to minimize the overall annual cost[Fan^{+}09]. The weight has been used to reflect the variance in load distribution and capacity. The following is the major differences between our study and theirs:

1. We study power plants which generate electricity, whereas their focus is substations which distribute electricity.

2. Our study determines the service scope of located power plants, whereas they determines the location of new substations and their capacities.

3. Our objective is to reduce energy loss in transmission, and theirs is to reduce the overall substation operation cost.

To our knowledge, this is the first attempt to reduce electricity loss in transmission by delimiting service scope for power plants. We have not found the information on the existing service scope of power plants within the U.S., so we are not able to compare our design with the existing service scope. We are also aware that the existing service scope may have taken other factors such as terrain, population density into consideration, whereas our model is idealized and simplified since it is only based on power plant location and capacity.

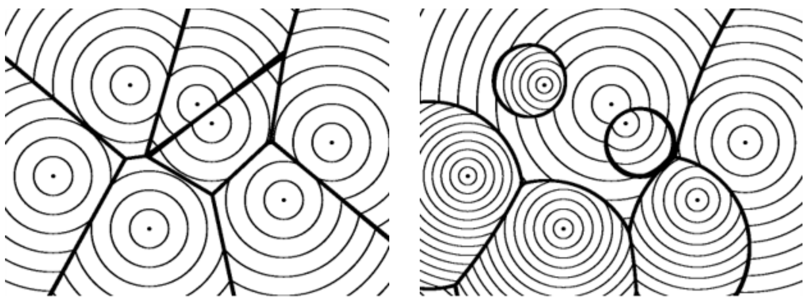
4 Methodology

Our goal is to generate and compare two Voronoi Diagrams. One is the traditional, unweighted VD. The other is the weighted VD, taking into account the difference of electricity production capacity among power plants. We expect to see power plants of larger capacity to have larger cells than which on the unweighted VD.

To incorporate weights into computation, weighted VD commonly uses different distance metrics other than Euclidean distance. We will discuss in detail regarding to the types of distance metrics and our choice for the project.

4.1 Distance Metrics and Weighting Schemes

As previously introduced in section 2.1, the Voronoi cell of a site p_i consists of a set of points x's that are closer to p_i than to any other site. In the context of traditional, unweighted VD, the “closeness” is usually measured by Euclidean distance, denoted as $d(x, p_i) = \|x - p_i\|$. However, the measure of “closeness”, or the distance metrics used to compute $d(x, p_i)$ can vary depending on application needs. The weighting schemes of VD is the use of modification on distance metrics to reflect the influence of weights. There are three types of weighting schemes that seemingly cater to our needs: multiplicatively weighted VD, additively weighted VD and the power diagram.



(a) (b)

Figure 10: VD for a set of sites with (a) no weighting (b) multiplicatively weighted[Let07].

In a multiplicatively weighted VD, the distance between a point x and a site p_i is defined as the Euclidean distance between them divided by the weight w_i[OBS09]:

$$d_{mw}(x, p_i) = \frac{\|x - p_i\|}{w_i}, w_i > 0 \tag{1}$$

For any Voronoi site, the larger the weight it is assigned, the larger the cell it will have. For a point on the borderline of two regions, its distance to one site is a multiple of the distance to the other, and the ratio is determined by the weights of two sites. Conceptually, multiplicative weighting is more motivated by the application. However, a multiplicatively weighted VD often contains non-convex regions, as well as enclave and exclave regions[CAZ19]. Moreover, the diagram may have regions that are not simply connected, as Figure 12b shows. Therefore, this approach might produce non-intuitive and impractical result in our application.

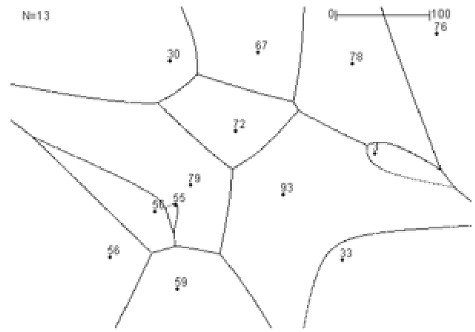
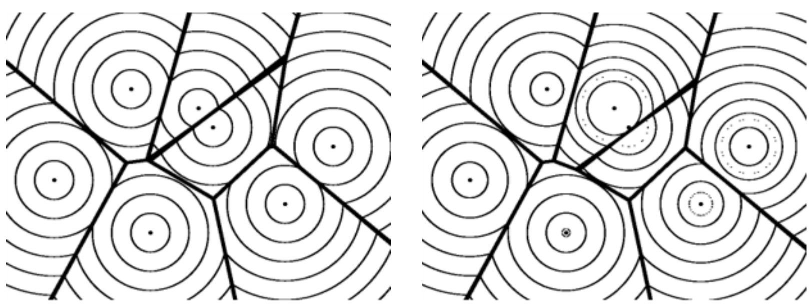


Figure 11: An additively weighted VD[SS06].

In an additively weighted VD, the distance between a point x and a site p_i is defined as the Euclidean distance between them subtracted by the weight w_i[OBS09]:

$$d_{mw}(x, p_i) = \|x - p_i\| - w_i, w_i > 0 \tag{2}$$

Similarly, for any Voronoi site, the larger the weight it is assigned, the larger the cell it will have. Additively weighted VD is simply connected and there is no enclave or exclave regions[CAZ19]. However, its edges can be arcs of hyperbolas, which might be more challenging for real world partitioning.



(a) VD without weighting (b) power diagram

Figure 12: Comparison of VD without weighting and power diagram[Let07].

Power diagram(PD) is in fact a variation of additively weighted VD. Its distance metric is defined as follows[OBS09] :

$$d_p(x, p_i) = \sqrt{\|x - p_i\|^2 - w_i} \tag{3}$$

This distance metric is also called power distance, usually defined on a set of circles. Each site p_i is inflated into a circle with radius of $\sqrt{w_i}$. In Figure 13, for example, the power distance between a point P and some site O is a segment PT tangent to the circle centered at O. Though the motivation of power diagram is not as obvious[Let07], its advantage is evident as it shares most nice properties with traditional VD – the cells are guaranteed to be convex, simply connected and have straight-line edges[CAZ19]. Therefore, we choose power diagram to delimit the service scope of power plants.

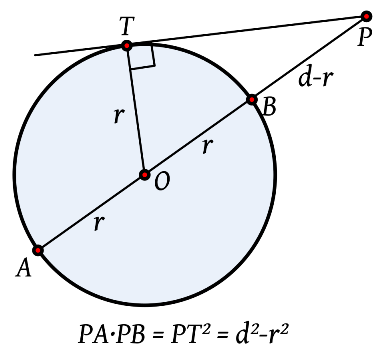


Figure 13: The distance metric of power diagram[Wik21].

4.2 Data Description and Preprocessing

The data used for the application is obtained from a comprehensive, open source database of the power plants all over the world. The database is developed by World Resources Institute (WRI) in partnership with Google Earth Outreach and multiple authoritative organizations and it mainly draws upon trusted sources such as national governments and other official sources, integrated with a small portion of crowdsourced data such as satellite images for accuracy improvement[Bye+19]. More importantly, among all current existing databases on this topic, it's the only one "truly comprehensive and fully accessible"[Bye+19] with all sources traceable to publicly available webpages, and is still under maintenance. The dataset in CSV form can be accessed at https://www.kaggle.com/eshaan90/global-power-plant-database. We adapt the latest official release from 2018.

The dataset contains over 28,500 power plants in 164 countries, representing about 80% of the world's capacity[Bye+19]. The set of attributes of power plants covers identification information (name and unique identifier), geographic information (country, longitude and latitude in WGS84 standard), electricity generation data (generation capacity in megawatts, actual generation for each year from 2013 to 2016, estimated annual electricity generation), and other miscellaneous information including owner of the plant, year of operation, attribution of data sources.

We leverage Python pandas library for exploratory data analysis and wrangling. Since our study object is the power plants within the United States, we only keep the dataset entries that are within the U.S. The processed dataset has 8119 instances with 5 attributes: ID, name, longitude, latitude and capacity. Through the data profiling, there are no missing data, and have reasonable range of numbers. The electrical generation capacity ranges from 1 to 6800 megawatts, the longitude ranges from -171.71 to 144.90, the latitude ranges from 13.30-71.29. All figures are within reasonable ranges.

4.3 Tools and Packages

We construct the traditional VD by using D3-Voronoi package and visualize by Google Maps JavaScript API. D3-Voronoi is a package written in JavaScript for computing 2D-VD. We choose D3-Voronoi because it implements Fortune's algorithm and also has decent visualization (e.g. supports zoom transitions)[Riv21]. The shortcoming is that the package itself does not support constructing more complex VD, including weighted VD, but it is satisfactory enough for constructing the traditional VD. Google Maps JavaScript API supports different types of map creation (roadmap, satellite, hybrid, and terrain), which enables display on web pages and mobile devices[Goo21a].

The power diagram is computed using the Computational Geometry Algorithms Library (CGAL), which is a C++ library of robust geometry algorithms. The power diagram is produced by constructing its dual diagram, a Regular triangulation. Then, we use Processing, a graphical library and IDE, to visualize the diagram computed by CGAL. We use Google Static Map API to obtain static map image as the canvas for drawing.

5 Results and Analysis

In this section, we are going to compare the service scope delimitation produced by traditional VD and by the weighted VD (power diagram) and seek a more reasonable and scientific solution.

5.1 Traditional Voronoi Diagram and Visualization

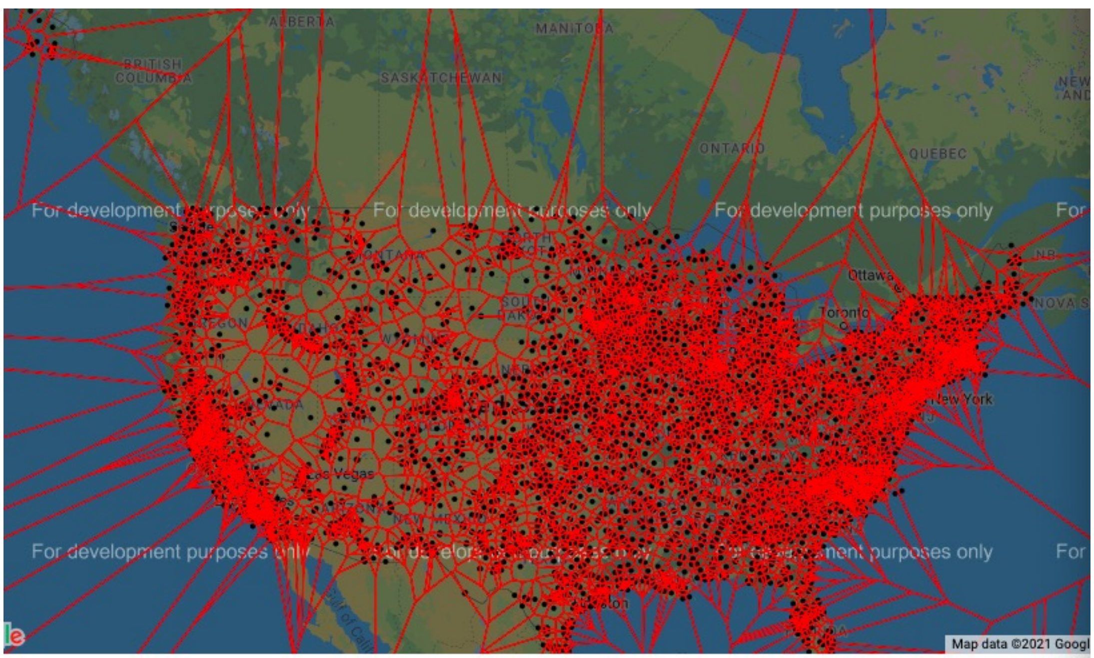


Figure 14: Service Scope Delimitation Visualization by Traditional VD

Gaining insight from[Shi21] on the usage of D3-Voronoi, traditional VDs are generated-shown in Figure 14 and 15. As we can see, the territory of United States is partitioned into small cells by the red lines, and within each cell there is one black dot, which represents a power plant. The area covered by each cell is the optimal service scope for the power plant in the cell.

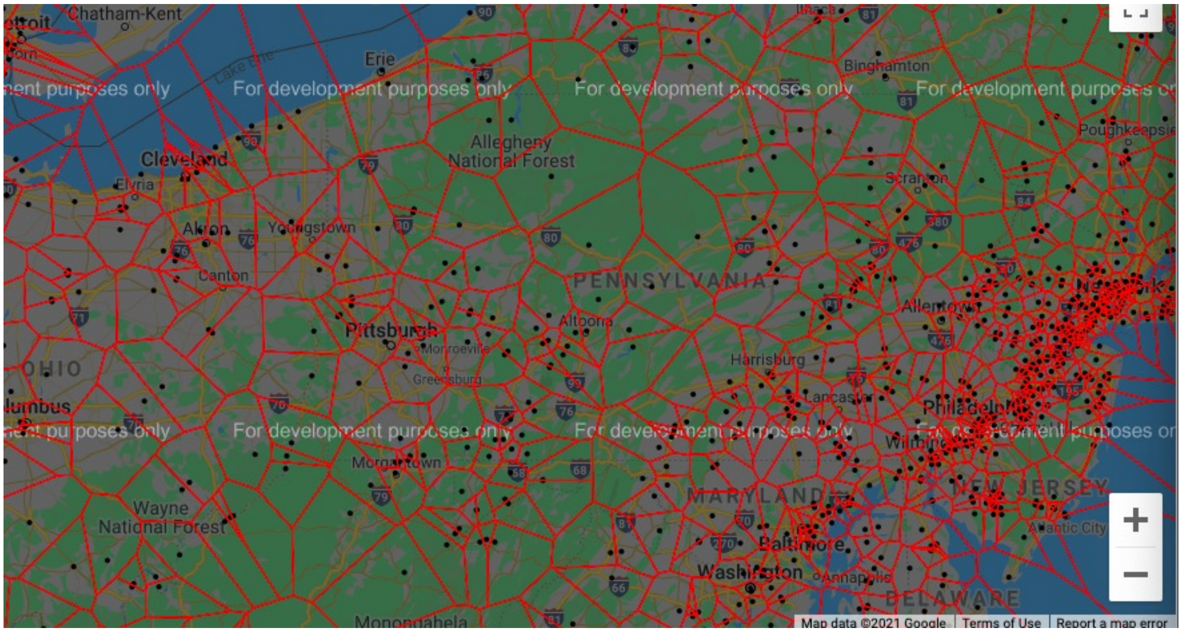


Figure 15: Service Scope Delimitation Visualization by Traditional VD (zoomed-in)

This approach simply "assigns" every point in the territory to the closest power plant measured by Euclidean distance, regardless of the different attributes the power plants have in reality. By the property of traditional VD (refer to property 5 in section 2.1.1), each edge has the same distance to the two adjacent sites.

5.2 Power Diagram and Visualization

5.2.1 Weight Tuning

We incorporate the capacities of the power plants as weight to construct the power diagram. The capacities of the power plants ranging from 1 to 6809 megawatts. The mean capacity is about 711 megawatts, and the median is about 12 megawatts. Thus, the data not only has a wide range, but is also heavily right-skewed. Therefore, we take the natural logarithm to the capacity values and then apply the min-max normalization, and now the weight ranges from 0 to 1 with mean = 0.32 and median = 0.28.

5.2.2 Visualization and Comparison

We construct PD on the scope of the entire U.S. territory. Figure 16 is the zoomed-in visualization that shows the delimitation of an area centered around the Grand Coulee Dam (GCD) power plant, and we will focus on that area for analysis. For comparison, Figure 17 is the partition done by traditional VD in the same area. GCD is the largest power plant in the U.S. with the largest capacity of 6,809 megawatts. Due to its large capacity, a more reasonable delimitation should assign it a larger service scope than which delimited by the traditional VD. Let the site of GCD be g, cell of GCD be c. Unlike the traditional VD, for each adjacent site p of g and the corresponding edge e in between, the distance from p to e is no longer the same as which from e to g; rather, the distance from g to e is significantly longer than e to p. For example, as shown in Figure 16, the site of Chief Joseph power plant has weight = 0.361 while g has weight = 1.000, and the distance from the site of Chief Joseph to the edge between it and g is 118.89 pixels in the diagram (equivalent to 118.89 pixel *390 meters/pixel = 46,367.1 meters[1]), whereas the distance from g to the edge is 123.39 pixels(equivalent to 123.39 pixel*390 meters/pixel = 48,122.1 meters). Since all edges of the c displays the similar pattern in PD, that is, further away from g than from the adjacent p, the area of c is larger than the corresponding cell of the traditional VD. Generalized to the whole diagram, for any two adjacent sites, the edge in between is closer to the one with larger weight. Since the area of a cell is determined by the distances from the edges to the site, the sites with larger weights (compared with the adjacent sites) have larger cells. Therefore, since power plants with larger capacities are assigned with larger service scopes, PD produces a more reasonable service scope delimitation.

[1]Google Map has zoom levels from 0 (most zoomed out) to 21 with 256x256 pixels tiles[Goo21b]. Here the diagram has zoom level = 8. Each zoom level has a conversion rate from pixel distance to the real world distance given the latitude of the location. With the latitude of the location of GCD = 47.96N, the estimated conversion rate is about 390 meters/pixel at zoom level = 8.[Goo21c]

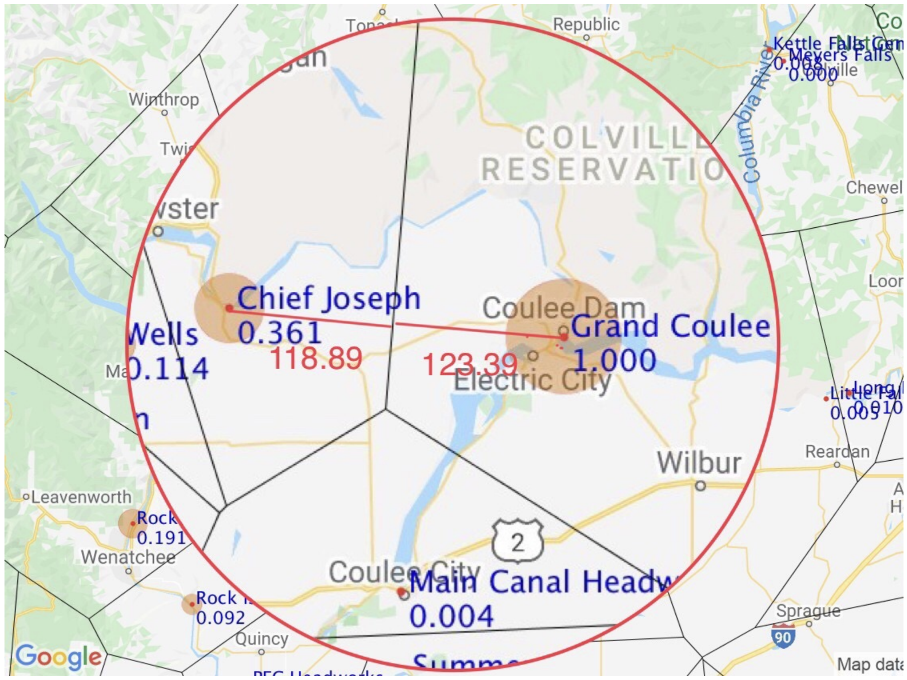


Figure 16: PD in GCD area: Chief Joseph power plant is 118.89 pixel distance away from the edge on its right, and GCD is 123.39 pixel distance away from the edge on its left.

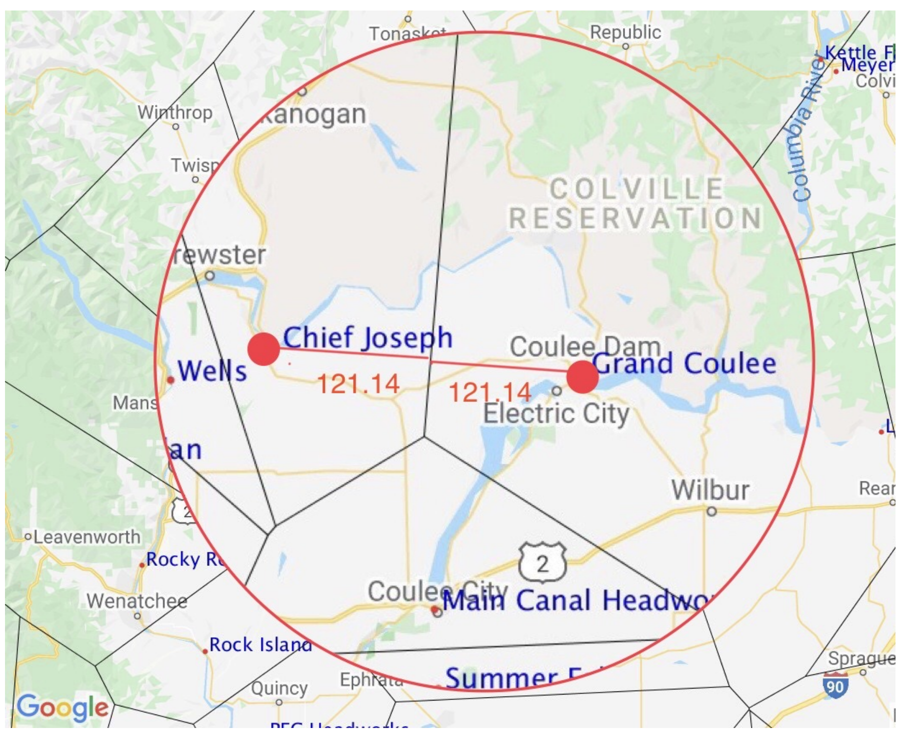


Figure 17: Traditional VD in GCD area: Chief Joseph power plant and GCD are both 121.14 pixel distance away from the edge between them.

6 Summary and Future Direction

This paper applies Voronoi diagram, both the traditional and the weighted VD (power diagram), to delimit service scopes for power plants in the U.S. to optimize transmission efficiency. The traditional VD can optimize transmission efficiency since each demand point is assigned to the closest power plant; however, it does not consider the huge variation in the capacity of the power plants, which might result in short supply or oversupply in reality. The weighted VD incorporates capacity as weight, so the service scope now is co-determined by both location and capacity. In this way, power plants with larger capacities are likely to have larger service scopes, so the model produced by weighted VD is more reasonable and scientific.

However, the model is still highly idealized. Besides capacity, future work could take more practical factors into consideration, including geographical unevenness and restrictions, demographical variations such as population density and electricity consumption per capita, etc.

References

[Age21] United States Environmental Protection Agency. *Centralized Generation of Electricity and its Impacts on the Environment.* https://www.epa.gov/energy/centralized-generation-electricity-and-its-impacts-environment. Mar. 6, 2021.

[Ale+13] Lyudmil Aleksandrov et al. "An Approximation Algorithm for Computing Shortest Paths in Weighted 3-d Domains". In: *Discrete Computational Geometry.* Springer, 2013, pp. 124–184. DOI: 10.1007/s00454-013-9486-0.

[Boi+02] Jean Daniel Boissonnat et al. "Triangulations in CGAL". In: *Computational Geometry* 22.1 (2002), pp. 5–19. DOI: 10.1016/S0925-7721(01)00054-2.

[Bye+19] Logan Byers et al. *A Global Database of Power Plants.* Tech. rep. https://www.wri.org/publication/global-power-plant-database. World Resources Institute, June 2019.

[CAZ19] Jacky Kit Cheung, Oscar Kin-Chung Au, and Kening Zhu. "Additive Voronoi Cursor: Dynamic Effective Areas Using Additively Weighted Voronoi Diagrams". In: *Human-Computer Interaction – INTERACT 2019.* Ed. by David Lamas et al. Springer International Publishing, 2019, pp. 273–292.

[Con21] Wikipedia Contributors. *Delaunay triangulation — Wikipedia, The Free Encyclopedia.* https://en.wikipedia.org/w/index.php?title=Delaunay_triangulation&oldid=1012384274. Mar. 6, 2021.

[Das+15] Joydeep Das et al. "Iterative Use of Weighted Voronoi Diagrams to Improve Scalability in Recommender Systems". In: *Advances in Knowledge Discovery and Data Mining.* Cham, Switzerland: Springer International Publishing, 2015, pp. 605–617.

[DO11] Stayan L. Devadoss and Joseph O'Rourke. *Discrete and Computational geometry.* Princeton University Press, 2011.

[Ele21] The Electropedia. *Energy Efficiency*. https://www.mpoweruk.com/energy_efficiency.html. Mar. 6, 2021.

[Epi21] UCLA Dept. of Epidemiology. *Dr. Snow's Report, "in Report on the Cholera Outbreak in the Parish of St. James, Westminster, during the Autumn of 1854"*. http://www.ph.ucla.edu/epi/snow/drsnowsrepparishstjames.html. Mar. 6, 2021.

[Fan+09] Yongfeng Fan et al. "The Dynamic Planning of Urban Substation Based on Weighted Voronoi Diagram". In: *2009 Asia-Pacific Power and Energy Engineering Conference*. Mar. 2009, pp. 1–4. DOI: 10.1109/APPEEC.2009.4918879.

[Flo21] Florian Rudolf. *Topological Operations*. https://www.iue.tuwien.ac.at/phd/rudolf/node24.html. Mar. 6, 2021.

[FM18] Xin Feng and Alan Murray. "Allocation Using a Heterogeneous Space Voronoi Diagram". In: *Journal of Geographical Systems* 20 (July 2018). DOI: 10.1007/s10109-018-0274-5.

[For86] Steven Fortune. "A Sweepline Algorithm for Voronoi Diagrams". In: *Proceedings of the Second Annual Symposium on Computational Geometry*. Yorktown Heights, New York, NJ: Association for Computing Machinery, 1986, pp. 313–322. DOI: 10.1145/10515.10549.

[GK12] Nicolai Guth and Philipp Klingel. "Demand Allocation in Water Distribution Network Modelling – A GIS-Based Approach Using Voronoi Diagrams with Constraints". In: *Application of Geographic Information Systems*. Ed. by Bhuiyan Monwar Alam. Rijeka: IntechOpen, 2012. Chap. 15. DOI: 10.5772/50014.

[Goo21a] Google. *Google Maps JavaScript API Overview*. https://developers.google.com/maps/documentation/javascript/overview. Mar. 6, 2021.

[Goo21b] Google. *Map and Tile Coordinates*. https://developers.google.com/maps/documentation/javascript/coordinates. Mar. 6, 2021.

[Goo21c] Google. *Zoom Level*. https://docs.mapbox.com/help/glossary/zoom-level/. Mar. 6, 2021.

[JRG11] Lili Ju, Todd Ringler, and Max Gunzburger. "Voronoi Tessellations and Their Application to Climate and Global Modeling". In: vol. 80. Feb. 2011, pp. 313–342. ISBN: 978-3-642-11639-1. DOI: 10.1007/978-3-642-11640-7_10.

[Kan08] James M. Kang. "Voronoi Diagram". In: *Encyclopedia of GIS*. Ed. by Shashi Shekhar and Hui Xiong. Boston, MA: Springer US, 2008, pp. 1232–1235. DOI: 10.1007/978-0-387-35973-1_1461.

[Kol04] Cyrus Kolahdouzan Mohammad afnd Shahabi. "Voronoi-Based K Nearest Neighbor Search for Spatial Network Databases". In: *Proceedings of the Thirtieth International Conference on Very Large Databases*. Vol. 30. Toronto, Canada: VLDB Endowment, 2004, pp. 840–851.

[KRE06] Heidi Koivistoinen, Minna Ruuska, and Tapio Elomaa. "A Voronoi Diagram Approach to Autonomous Clustering". In: *Discovery Science*. Ed. by Ljupčo Todorovski, Nada Lavrač, and Klaus P. Jantke. Berlin, Heidelberg: Springer Berlin Heidelberg, 2006, pp. 149–160.

[Let07] D. Letscher. "Vector Weighted Anisotropic Voronoi Diagrams and Delaunay Traingulations". In: *CCCG*. Ontario, Canada, 2007.

[Li+12] Hui Li et al. "Spatial Modeling of Bone Microarchitecture". In: *Three-Dimensional Image Processing (3DIP) and Applications II*. Ed. by Atilla Baskurt and Robert Sitnik. Vol. 8290. International Society for Optics and Photonics. SPIE, 2012, pp. 232–240. DOI: 10.1117/12.907371.

[LLT11] Ickjai Lee, Kyungmi Lee, and Christopher Torpelund-Bruin. "Raster voronoi tessellation and its application to emergency modeling". In: *Geo-spatial Information Science* 14.4 (Dec. 2011), pp. 235–245. DOI: 10.1007/s11806-011-0569-x.

[LSL16] Jingna Liu, Xiaoyun Sun, and Shujuan Liu. "Weighted Node Network Voronoi Diagram and its application to optimization of chain stores layout". In: 7 (2016), pp. 679–688. DOI: 10.1007/s13042-015-0491-x.

[Mar19] Jeremi Martin. *Distributed vs. Centralized Electricity Generation: Are We Witnessing a Change of Paradigm? An Introduction to Distributed Generation Executive Summary Content*. Tech. rep. HEC Paris, May 2019.

[OBS09] Arsuyuki Okabe, Barry Boots, and Kokichi Sugihara. *Spatial Tessellations: Concepts and Applications of Voronoi Diagrams*. Wiley Series in Probability and Statistics. Wiley, 2009.

[Res21] ResearchGate. *Developing a Scene-Based Triangulated Irregular Network (TIN) Technique for Individual Tree Crown Reconstruction with LiDAR Data - Scientific Figure on ResearchGate*. https://www.researchgate.net/figure/Comparison-of-Delaunay-and-non-Delaunay-triangulations-a-Delaunay-triangulation-b_fig2_338141441. Mar. 6, 2021.

[Riv21] Philippe Rivière. *D3-Voronoi*. https://github.com/d3/d3-voronoi. Mar. 6, 2021.

[Sch98] Alfred H. Schumann. "Thiessen Polygon". In: *Encyclopedia of Hydrology and Lakes*. Dordrecht, Netherlands: Springer Netherlands, 1998, pp. 648–649. DOI: 10.1007/1-4020-4497-6_220.

[Shi21] Shimizu. *Takasaki City Fuel Gas Station Voronoi Diagram*. https://gist.github.com/shimizu/1f8c1f44e29f93a34a2be3be1db5829d. Mar. 6, 2021.

[Shm21] Shmoon. *Focus and Directrix*. https://www.shmoop.com/conics/parabola-parts.html. Mar. 6, 2021.

[SPR19] Francesco Santelli, Francesco Palumbo, and Giancarlo Ragozini. "Statistical Archetypal Analysis for Cognitive Categorization". In: *New Statistical Developments in Data Science*. Ed. by Alessandra Petrucci, Filomena Racioppi, and Rosanna Verde. Cham, Switzerland: Springer International Publishing, 2019, pp. 77–89.

[SS06] Mehdi Sharifzadeh and Cyrus Shahabi. "Additively Weighted Voronoi Diagrams for Optimal Sequenced Route Queries". In: (Jan. 2006).

[Swi21] Swiss Federal Institute of Technology in Zürich. *Delaunay Triangulations*. https://www.ti.inf.ethz.ch/ew/Lehre/CG13/lecture. Mar. 6, 2021.

[Tah06] Abdel Aziz Taha. "Coloring of the Self-Organizing Map based on Class Labels". MA thesis. Oct. 2006.

[Tra21] Khuyan Tran. *How to Find the Nearest Hospital with a Voronoi Diagram*. https://towardsdatascience.com/how-to-find-the-nearest-hospital-with-voronoi-diagram-63bd6d0b7b75. Mar. 6, 2021.

[Wan+20] Ajun Wan et al. "Study on spatial layout optimization of Linpan settlements based on point pattern analysis". In: *Environment, Development and Sustainability* (Oct. 2020). DOI: 10.1007/s10668-020-01042-z.

[Wik21] Wikipedia Contributors. *Power diagram — Wikipedia, The Free Encyclopedia*. https://en.wikipedia.org/w/index.php?title=Power_diagram&oldid=984639050. Mar. 6, 2021.

[Xin+10] Zheng Xinqi et al. "Urban Cluster Layout Based on Voronoi Diagram". In: *Advanced Techniques in Computing Sciences and Software Engineering*. Ed. by Khaled Elleithy. Dordrecht, Netherlands: Springer Netherlands, 2010, pp. 13–18.

[YJU12] Wilfredo F. Yushimito, Miguel Jaller, and Satish Ukkusuri. "A Voronoi-Based Heuristic Algorithm for Locating Distribution Centers in Disasters". In: *Networks and Spatial Economics* 12.1 (Mar. 2012), pp. 21–39. DOI: 10.1007/s11067-010-9140-9.

[Yu+20] Wenhao Yu et al. "Service Area Delimitation of Fire Stations with Fire Risk Analysis: Implementation and Case Study". In: *International journal of en-*

vironmental research and public health 17.6 (2020), p. 2030. DOI: 10.3390/ijerph17062030.

[ZJC18] Pengyan Zhang, Wenlong Jing, and Yunzeng Chen. "Weighted Voronoi Diagram-Based Simulation and Comparative Analysis of Ecosystem Service Coverage: Case Study of the Zhongyuan Urban Agglomeration". In: *Journal of Sensors* (2018). DOI: 10.1155/2018/7147524.